Impossibility of VDFs in the ROM

The Complete Picture

Max von Consbruch*

Joint work with Hamza Abusalah†, Karen Azari*, Chethan Kamath‡ and Erkan Tairi§

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Overview

Main Theorem. Verifiable delay functions do not exist in the random oracle model.

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This Talk:

- Random Oracle Model

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The (parallel, query-complexity) Random Oracle Model

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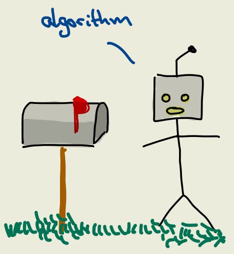

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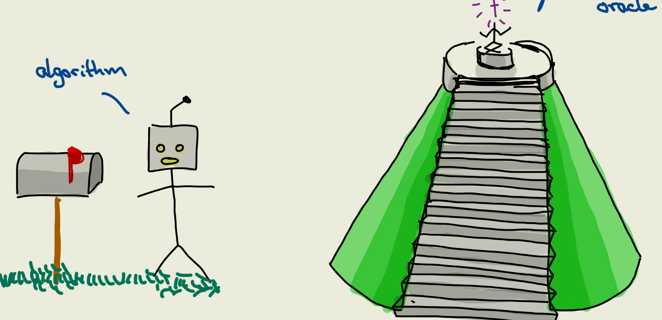

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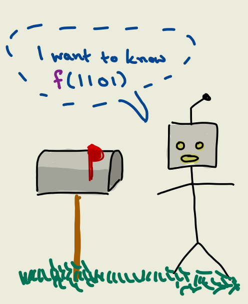

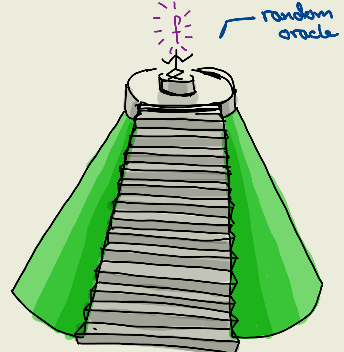

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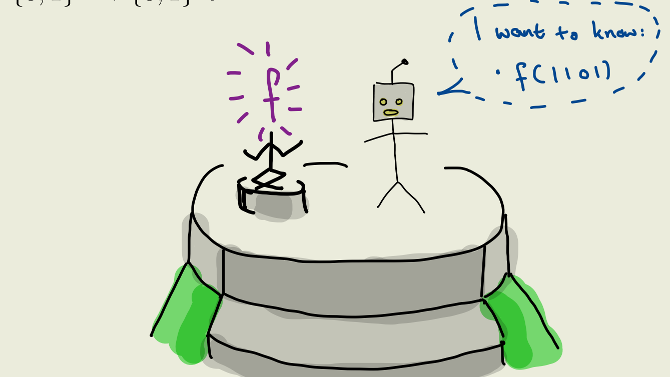

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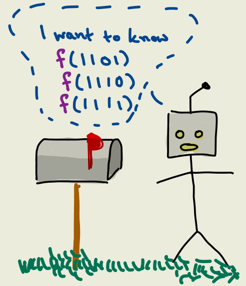

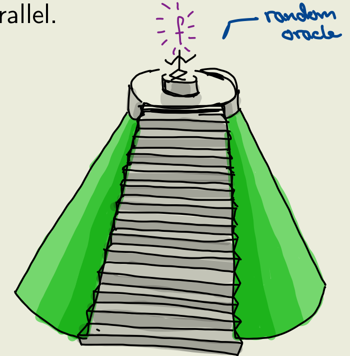

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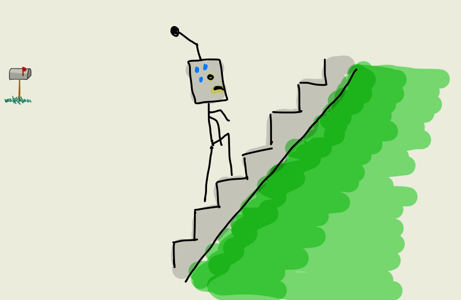

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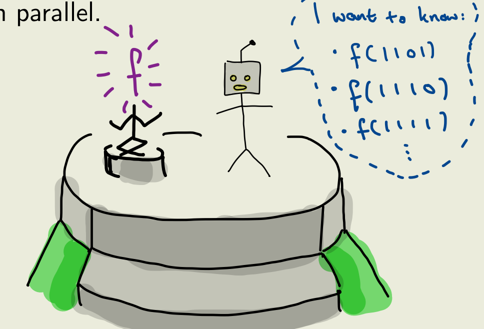

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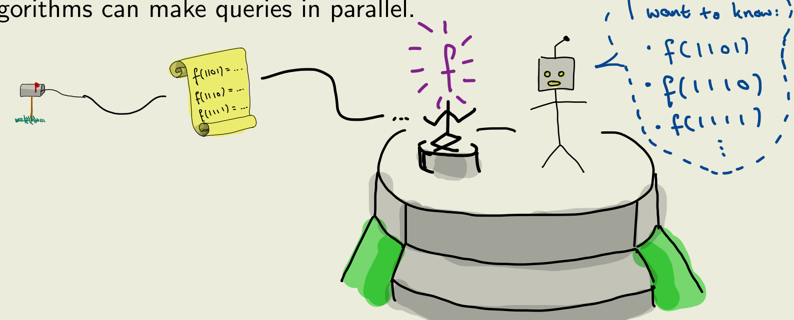

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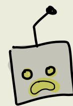
- Two measures for complexity:
    1. Query-Complexity ("how many queries in total?")
    2. Round-Complexity ("how many rounds of queries?")



ROM: Examples

Problem	Query Complexity	Round Complexity
Find $x \in \{0,1\}^*$ s.t. $f(x)$ has $\log_2(T)$ leading zeroes	T	1

$f(x) = \underbrace{00\cdots0}_{\log_2(T)}*\dots*$

ROM: Examples

Problem	**Query Complexity**	**Round Complexity**
Find $x \in \{0,1\}^*$ s.t. $f(x)$ has $\log_2(T)$ leading zeroes	T	1
Calculate $f^T(x)$ for given $x \leftarrow \{0,1\}^*$	T	T

$y = \underbrace{f(f(\ldots(f}_{T}(x))\ldots))$

Verifiable Delay Functions in the ROM

A *Verifiable Delay Function (VDF)* is a...

- delay function...

$$F : \mathcal{X} \to \mathcal{Y} \text{ can be evaluated in } T = \text{poly}(\lambda) \text{ rounds.}$$

I want to know F(x)!

Example: $F(x) = f^T(x) = f(f(\dots(f(x))\dots))$.

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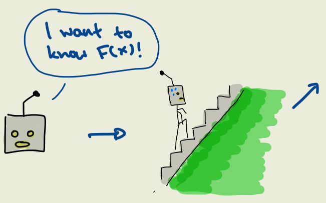

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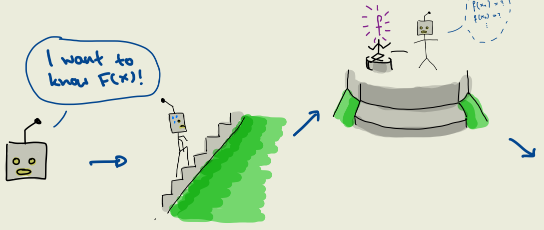

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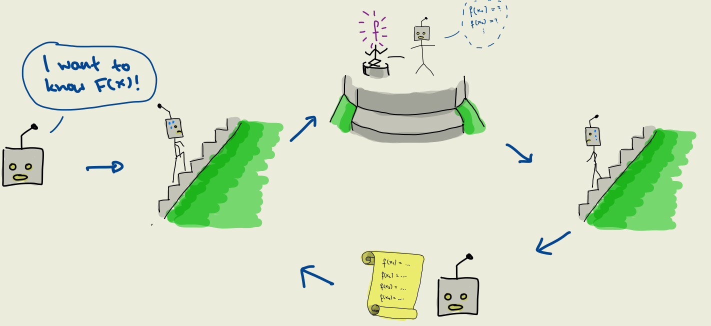

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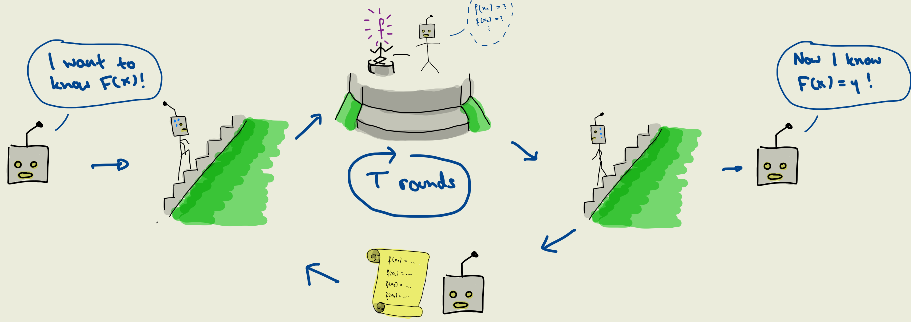

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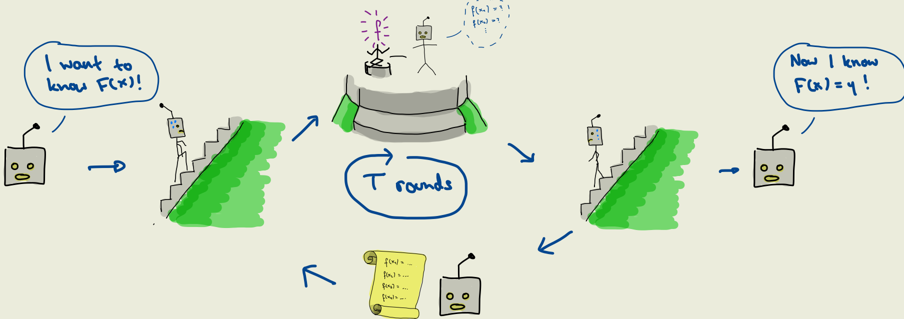

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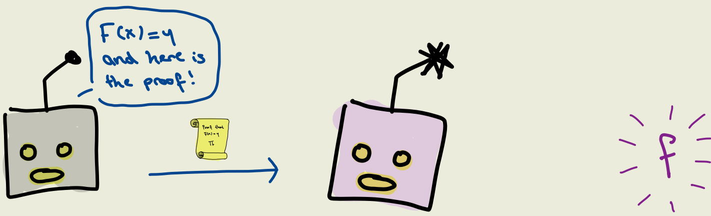

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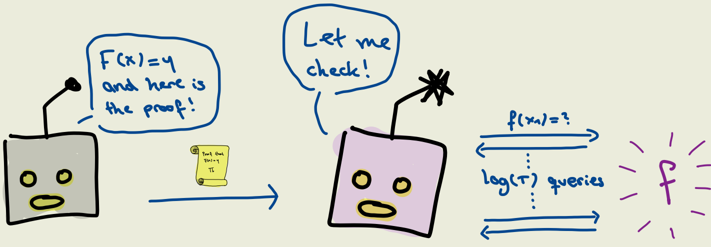

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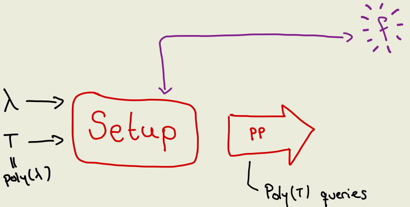

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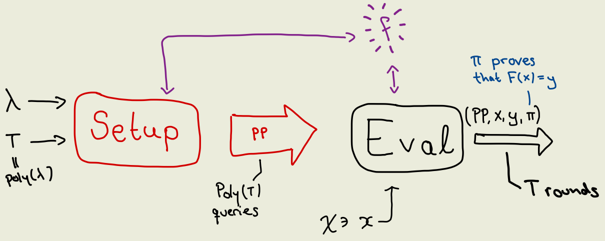

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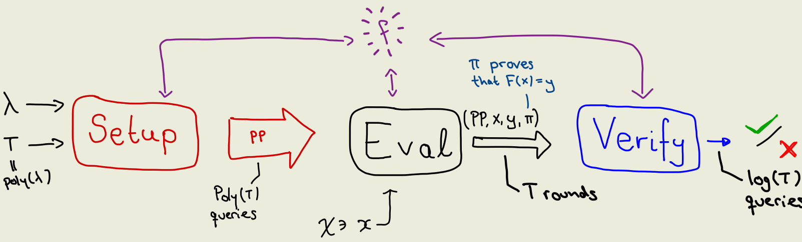

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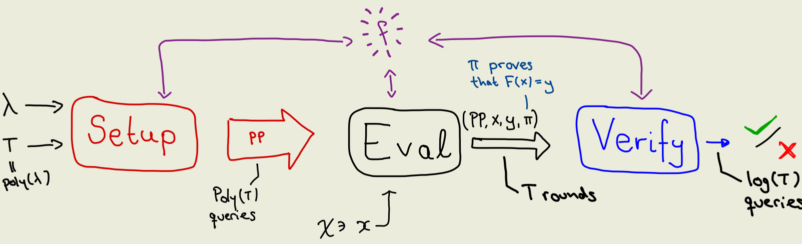

- **sequentiality:** it is impossible to evaluate F in $\ll T$ rounds.
- **soundness:** no polynomial-query algorithm can construct (x, y, π) with $y \neq F(x)$ and $\text{Verify}^f(\text{pp}, x, y, \pi) = 1$.

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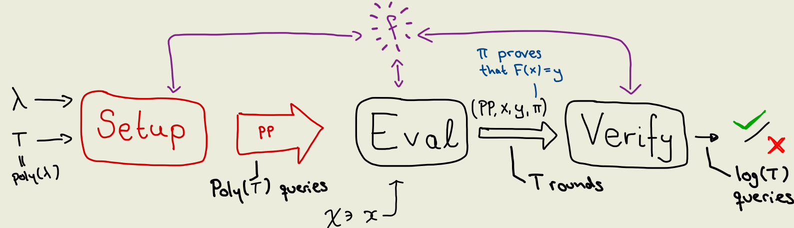

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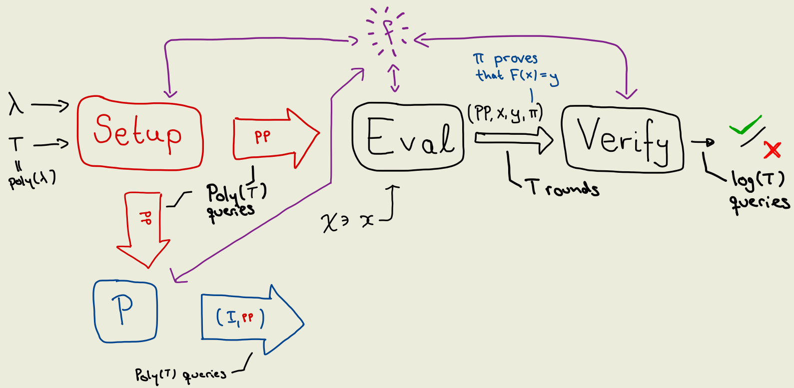

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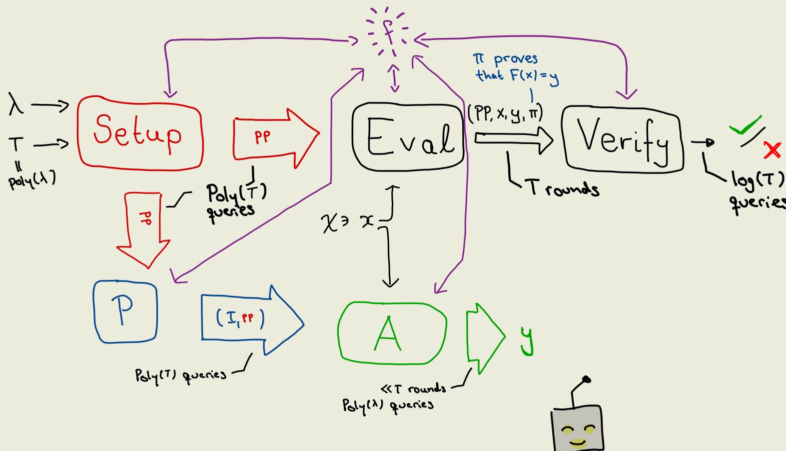

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[GRY25]	deterministic	usual	$O(\log(T))$

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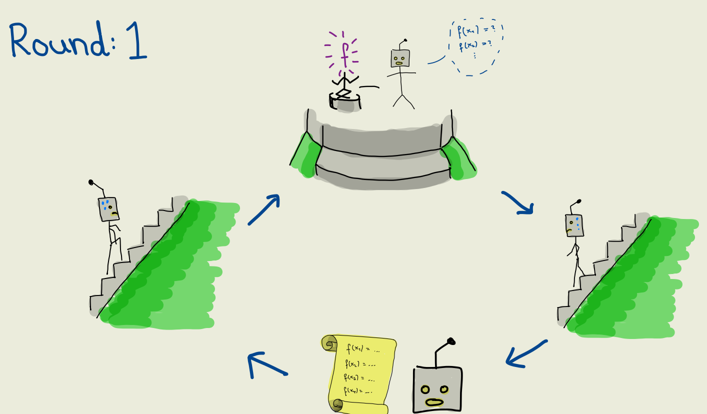
Breaking "tight" VDFs

Idea: Run $\mathrm{Eval}^f(\mathrm{pp}, x)$ as usual...



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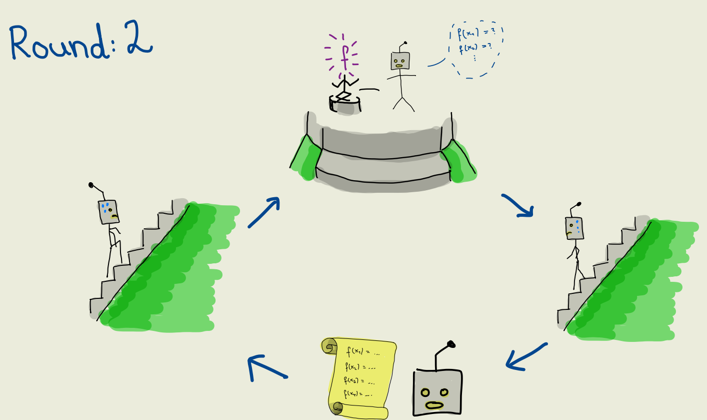

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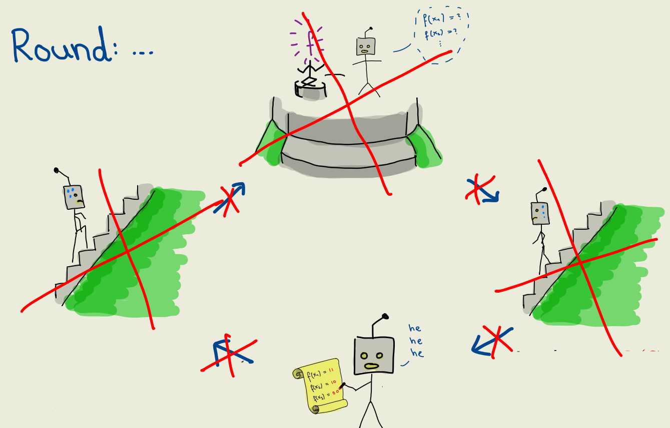

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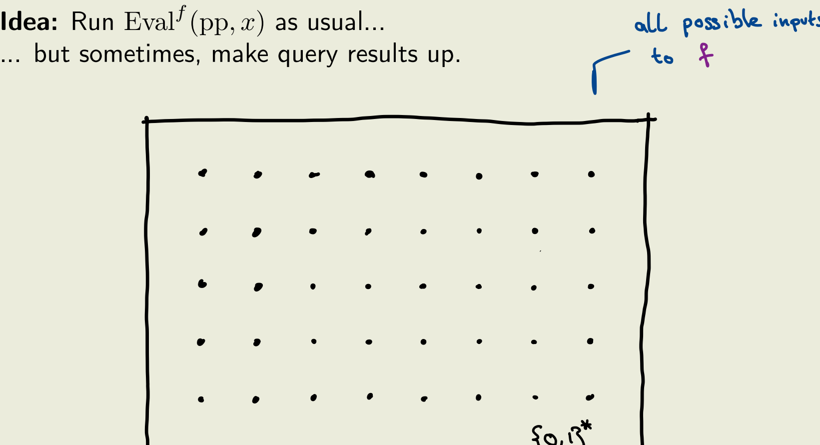

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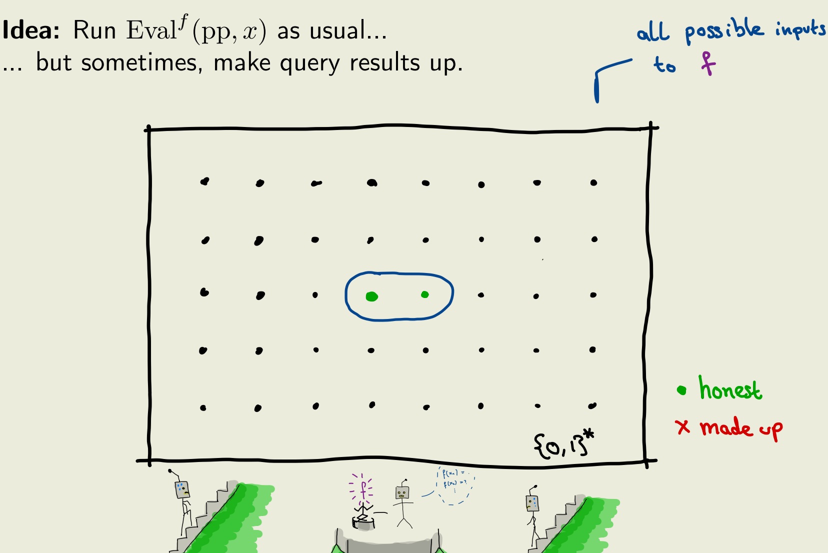

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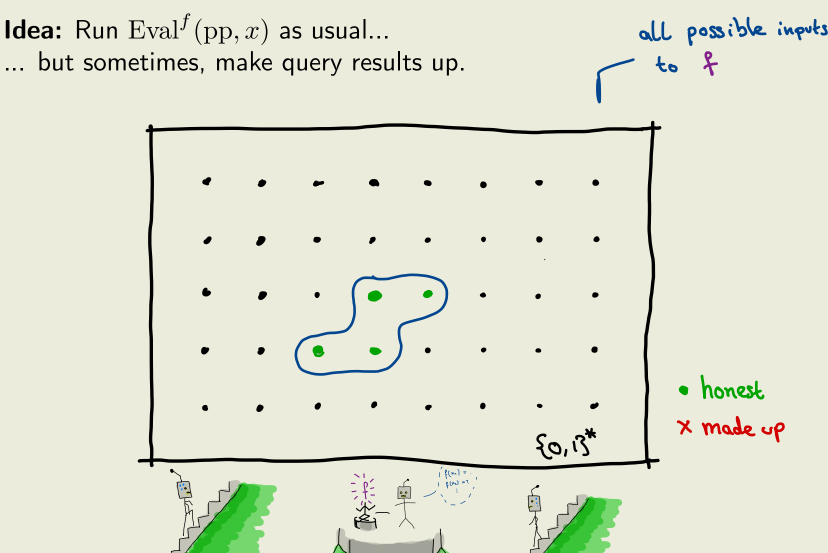

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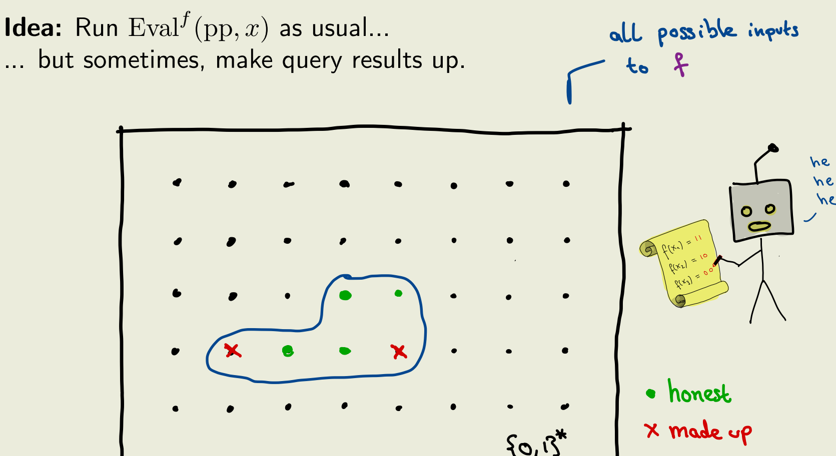

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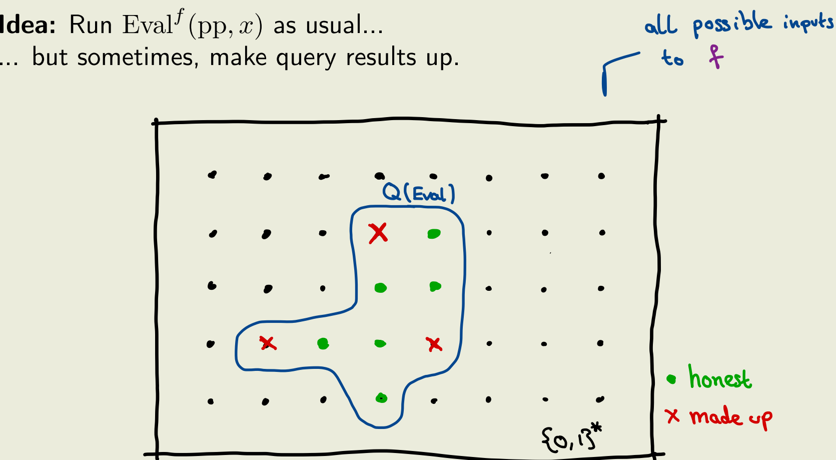

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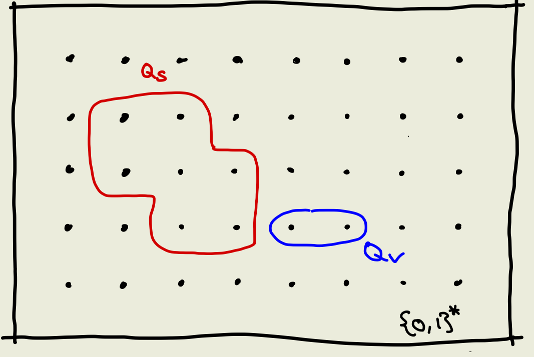

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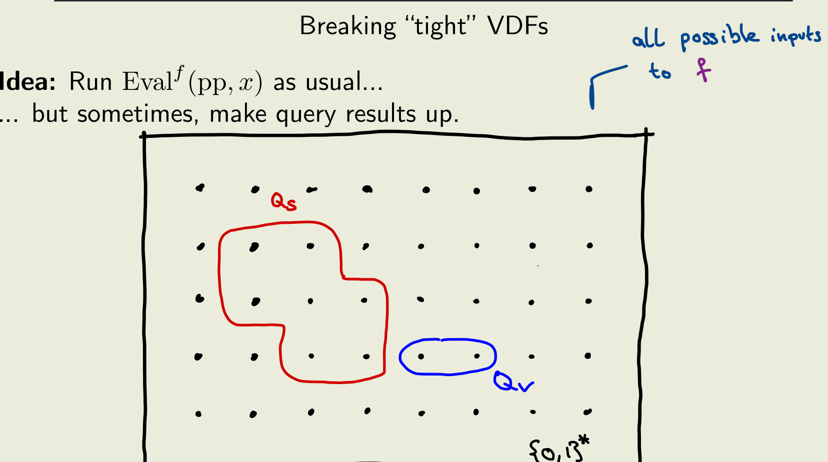

Main Observation: If none of the made up queries lie in $\color{red}{Q(\mathrm{Setup})}$ or $\color{blue}{Q(\mathrm{Verify})}$, then we win.

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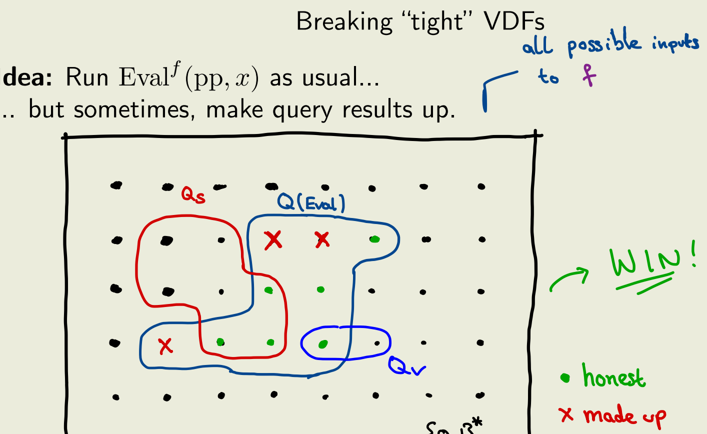

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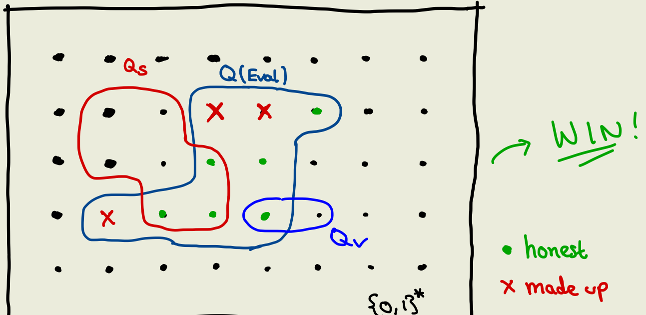
all possible inputs to f



Main Observation: If none of the made up queries lie in $Q(\mathrm{Setup})$ or $Q(\mathrm{Verify})$, then we win.
<u>Theorem.</u> With $q = \#(Q(\mathrm{Setup}) \cup Q(\mathrm{Verify}))$, we can evaluate F in $T - O(T/q)$ rounds.*

* with high probability

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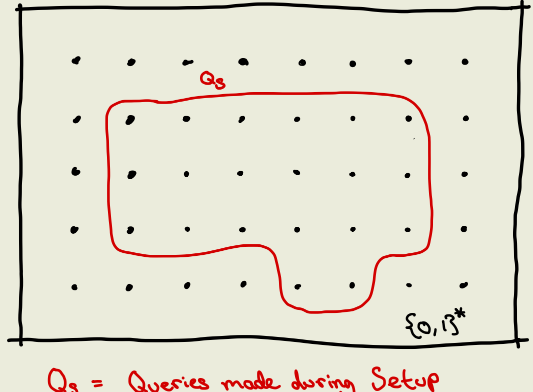
Theorem [DGMV20]. With $q = \#(Q(\mathrm{Setup}) \cup Q(\mathrm{Verify}))$, we can evaluate F in $T - O(T/q)$ rounds.

Problem: What if Setup makes a lot of queries?



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Solution: use preprocessing to simulate *honest* Eval runs!

Round: 1

Q_S

Q(Eval^f(x_1))

{0,1}*

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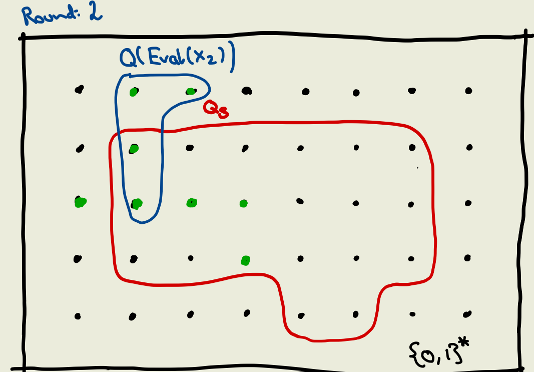

Attack 1: the Lazy Honest Evaluator [DGMV20]

Breaking "tight" VDFs

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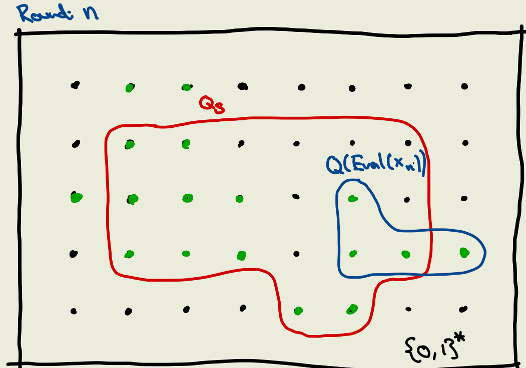

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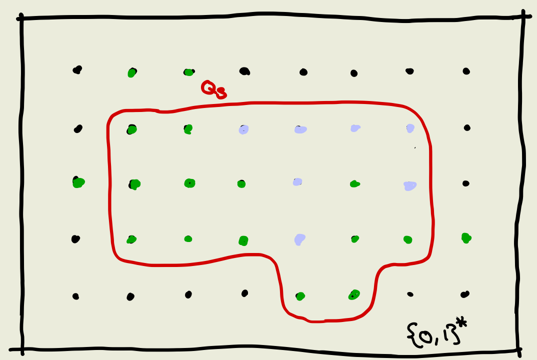

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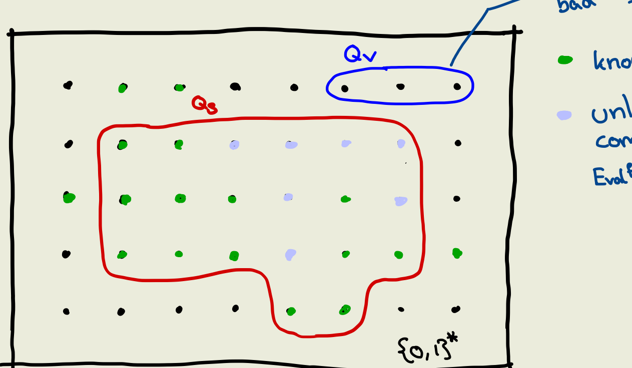

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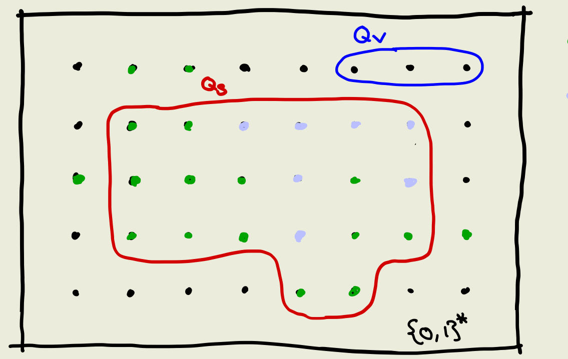


known

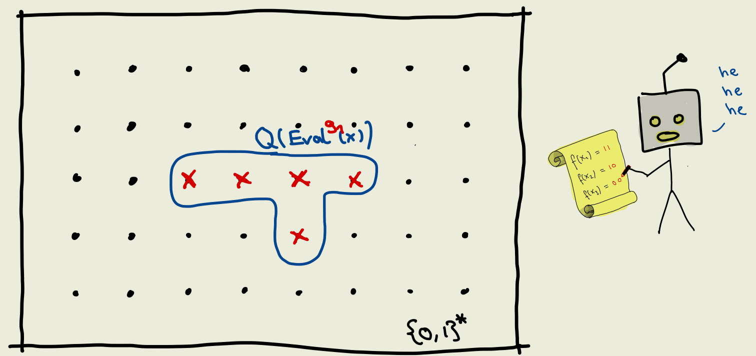
unlikely to come up in $\text{Eval}^f(x)$ for $x \leftarrow X$

Attack 2: the Dishonest Group; [MSW20]

Attacking perfectly sound VDFs

Idea:

- Run Eval, and make up all of the queries.



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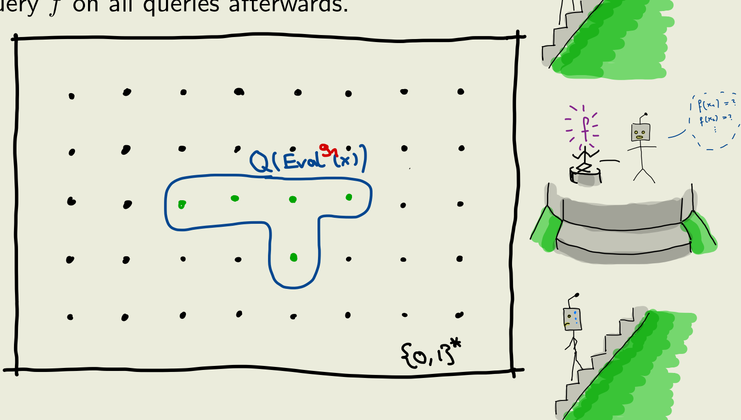

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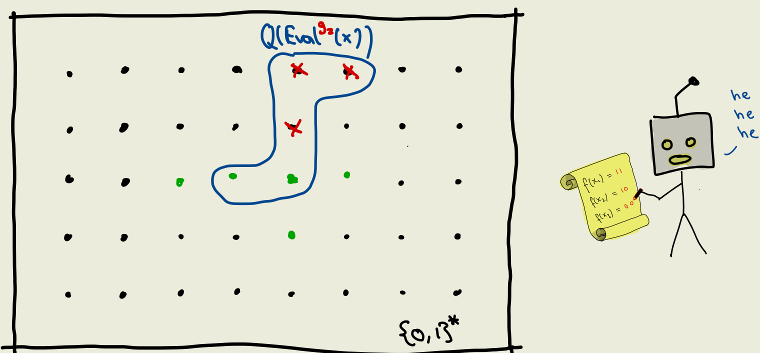

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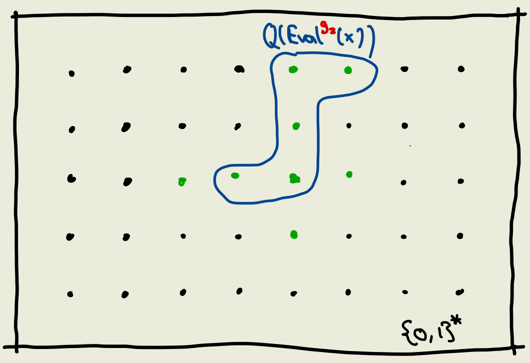

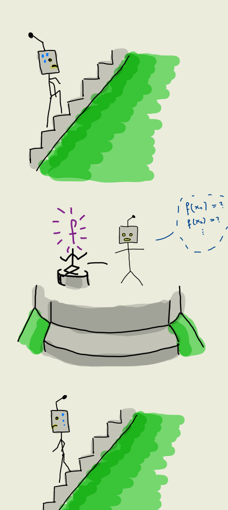

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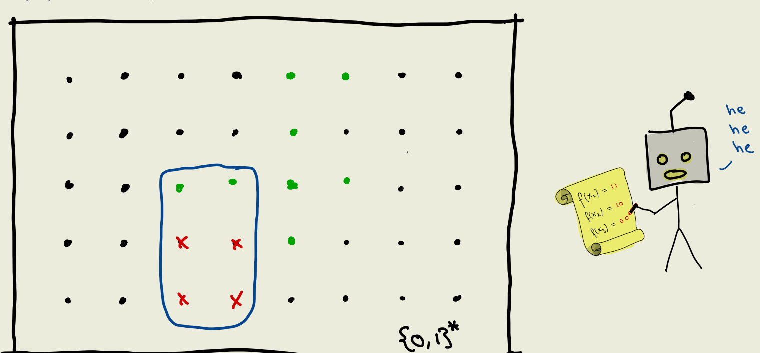

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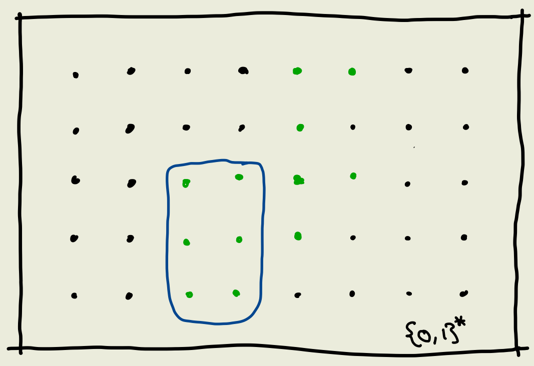

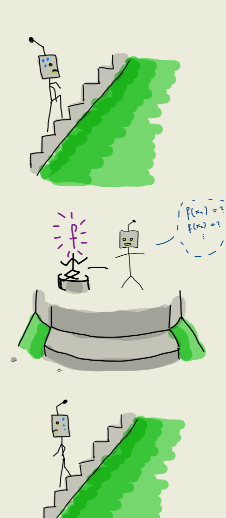

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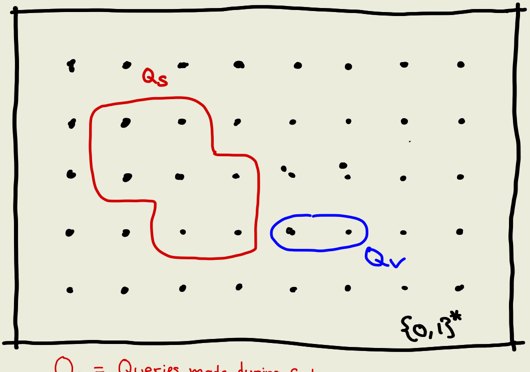

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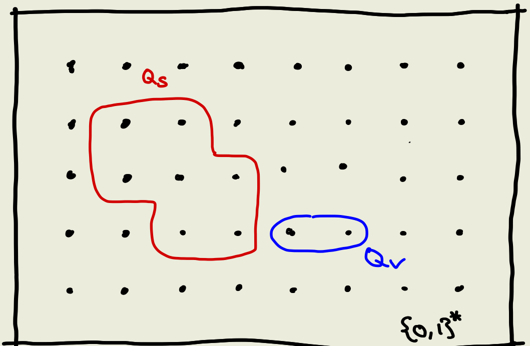

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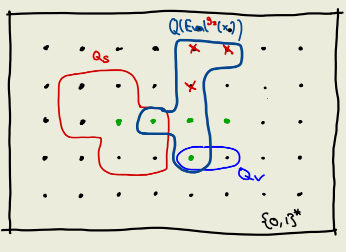
Observation: In each round, either win, or learn the value of f at a new input $x \in \color{red}{Q(\text{Setup})} \cup \color{blue}{Q(\text{Verify})}$.

two cases:



No made-up query in Q_S or Q_V:

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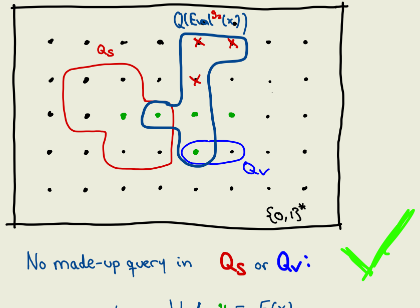

- correct result! $y = F(x)$

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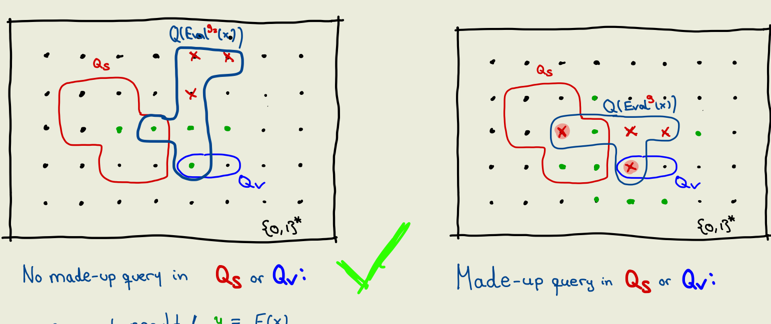

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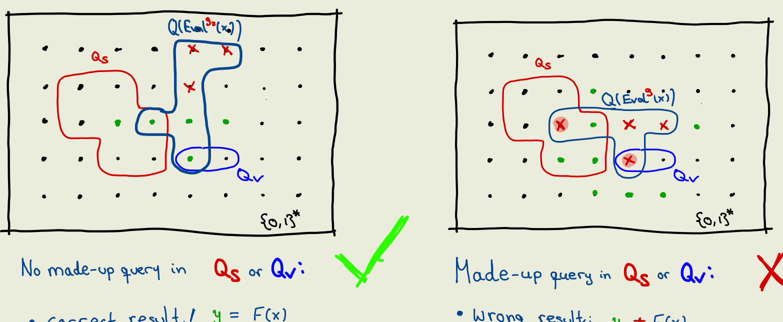

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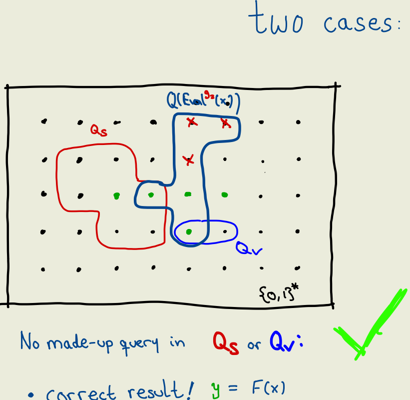

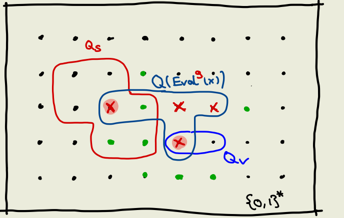

Made-up query in Qs or Qv:

• Wrong result: y ≠ F(x)

• learn ≥1 new important query

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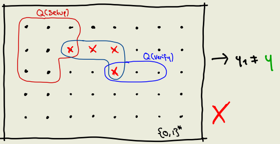

→ $y_1 \neq y$

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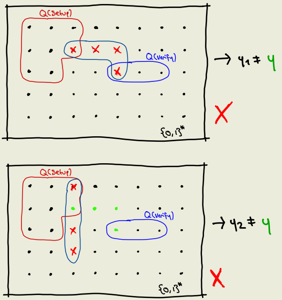

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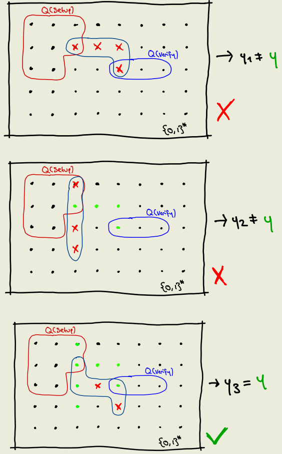

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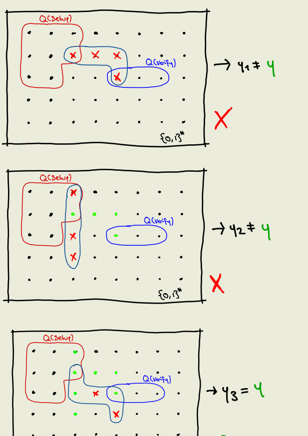

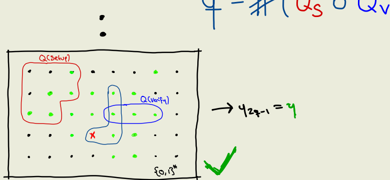

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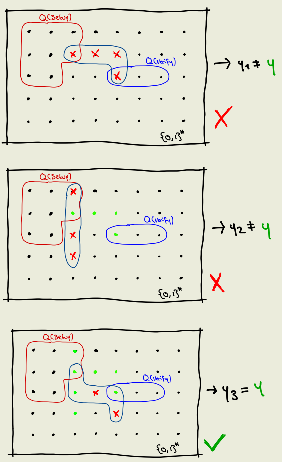

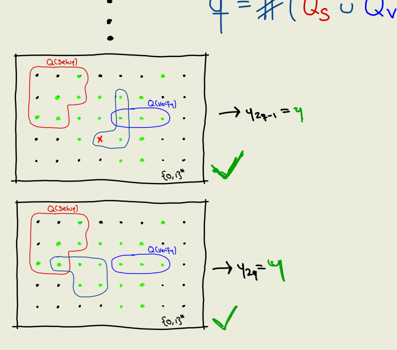

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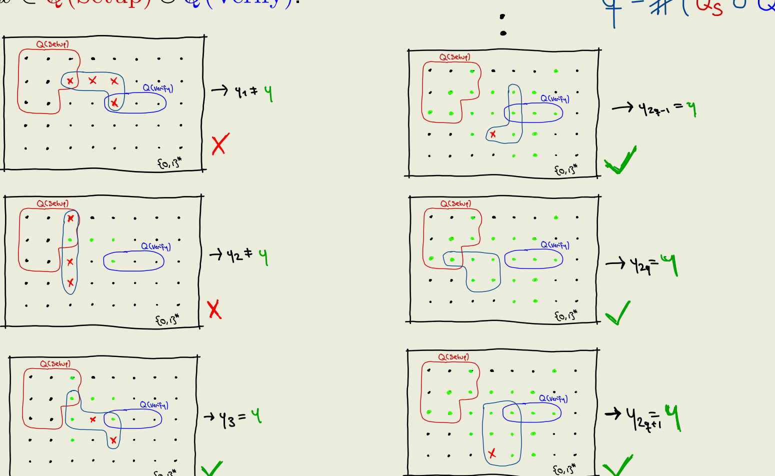

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Hence: If Setup makes $O(\log(T))$ queries, we break sequentiality in $O(\log(T))$ rounds.

Attack 3: the Extended Dishonest Group; [GRY25]

Attacking VDFs with deterministic Setup

Approach: construction of an algorithm $\mathsf{A}_{\mathrm{GRY}}$ that breaks soundness if $\mathsf{A}_{\mathrm{MSW}}$ fails to break sequentiality.

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Work	Setup	Soundness	# Rounds

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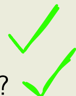

Our work: Positive answer to both questions!

Extending [MSW20] to Expensive Setup

Main Observation:

If $\mathsf{A}^{f}_{\mathrm{MSW}}(\mathrm{pp}, x)$ does not make a (new) query at $x \in Q(\mathrm{Setup})$, then it computes $y = F(x)$ correctly in $O(\#Q(\mathrm{Verify}))$ rounds.

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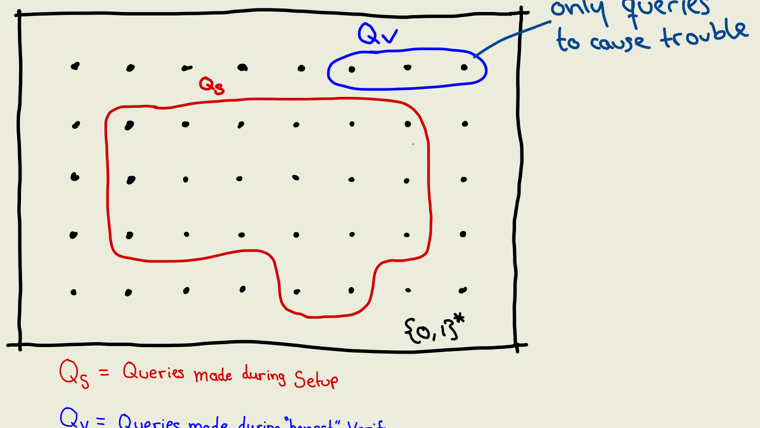

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We want $\mathsf{A}^f_{\mathrm{MSW}}(\mathrm{pp}, x, I)$ not to make queries to $Q(\mathrm{Setup})$.

Extending [MSW20] to Expensive Setup

Our Construction:

simulate executions of $\mathsf{A}^{f}_{\mathrm{MSW}}(\mathrm{pp}, x)$ for $x \leftarrow X$

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2. For $i = 1, \ldots, t$:
   - 2.1 Sample $x \leftarrow X$ at random
   - 2.2 Run $y \leftarrow \mathsf{A}^f_{\mathrm{MSW}}(\mathrm{pp}, x, I)$
   - 2.3 Store the results of all queries made in the previous step in I.
3. Return I.

This ensures that $\mathsf{A}^f_{\mathrm{MSW}}(\mathrm{pp}, x, I)$ (likely) does not make new queries to $Q(\mathrm{Setup})$.

Extending [MSW20] to Expensive Setup

Our Construction:

simulate executions of $\mathsf{A}^f_{\mathrm{MSW}}(\mathrm{pp}, x)$ for $x \leftarrow X$

$\mathsf{P}^f(\mathrm{pp})$:

1. Set $I = \emptyset$, sample $t \leftarrow [2q_s]$ at random
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this is what we wanted!

Overview of Results

Previously Open:

1. Can [MSW20] be extended to inefficient setup?
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Main Theorem. If Π_{VDF} is a sound VDF in the ROM, then it does not satisfy sequentiality.

Thank you!

Questions?

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